

Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

Variable a	Constant $a = a_c$
$a = \frac{dv}{dt}$	$v = v_0 + a_c t$
$v = \frac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$
$a ds = v dv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates	r, θ, z Coordinates
$v_x = \dot{x}$ $a_x = \ddot{x}$	$v_r = \dot{r}$ $a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$ $a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$ $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$ $a_z = \ddot{z}$	$v_z = \dot{z}$ $a_z = \ddot{z}$

n, t, b Coordinates

$v = \dot{s}$	$a_t = \dot{v} = v \frac{dv}{ds}$
	$a_n = \frac{v^2}{\rho}$ $\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{ d^2y/dx^2 }$

Relative Motion

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Rigid Body Motion About a Fixed Axis

Variable a	Constant $a = a_c$
$\alpha = \frac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \frac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha_c t^2$
$\omega d\omega = \alpha d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$$

Relative General Plane Motion—Translating Axes

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$$

Relative General Plane Motion—Trans. and Rot. Axis

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

KINETICS

Mass Moment of Inertia

$$I = \int r^2 dm$$

Parallel-Axis Theorem

$$I = I_G + md^2$$

Radius of Gyration

$$k = \sqrt{\frac{I}{m}}$$

Equations of Motion

Particle	$\Sigma \mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G \alpha$ or $\Sigma M_P = \Sigma (\mathcal{M}_k)_P$

Principle of Work and Energy

$$T_1 + U_{1-2} = T_2$$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force

$$U_F = \int F \cos \theta ds$$

Constant force

$$U_F = (F_c \cos \theta) \Delta s$$

Weight

$$U_W = -W \Delta y$$

Spring

$$U_s = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right)$$

Couple moment

$$U_M = M \Delta \theta$$

Power and Efficiency

$$P = \frac{dU}{dt} = \mathbf{F} \cdot \mathbf{v} \quad \epsilon = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{U_{\text{out}}}{U_{\text{in}}}$$

Conservation of Energy Theorem

$$T_1 + V_1 = T_2 + V_2$$

Potential Energy

$$V = V_g + V_e, \text{ where } V_g = \pm W y, V_e = +\frac{1}{2}ks^2$$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma \int \mathbf{F} dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma \int \mathbf{F} dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$$\Sigma(\text{sys. } m\mathbf{v})_1 = \Sigma(\text{sys. } m\mathbf{v})_2$$

Coefficient of Restitution

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$
	where $H_O = (d)(mv)$

Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G dt = (\mathbf{H}_G)_2$
	where $H_G = I_G\omega$
	$(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O dt = (\mathbf{H}_O)_2$
	where $H_O = I_O\omega$

Conservation of Angular Momentum

$$\Sigma(\text{sys. } \mathbf{H})_1 = \Sigma(\text{sys. } \mathbf{H})_2$$